MF02 MAGNETIC FORCE

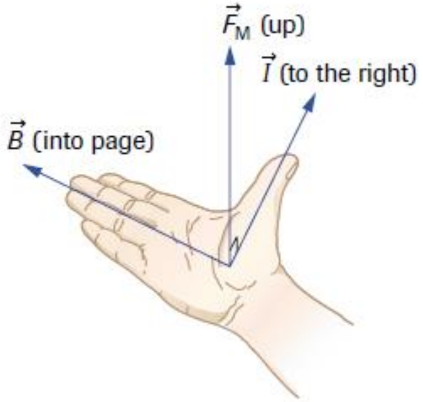
SPH4U

CH 8 (KEY IDEAS)

- define and describe concepts related to magnetic fields
- compare and contrast the properties of electric, gravitational, and magnetic fields
- predict the forces on moving charges and on a current-carrying conductor in a uniform magnetic field
- perform and analyze experiments and activities on objects or charged particles moving in magnetic fields
- analyze and explain the magnetic fields around coaxial cables
- describe how advances in technology have changed scientific theories
- evaluate the impact of new technologies on society

EQUATIONS

- Right-Hand Rule: Magnetic Force



- Magnetic Force on a Moving Charge

$$F_M = qvB \sin\theta$$

- Magnetic Force on a Conductor

$$F = IlB \sin\theta$$

MAGNETIC FORCE ON MOVING CHARGES

- The magnetic field produced by a current flowing through a wire can affect the magnetic field of a compass
  - A current can exert a force on a magnetic field
- Two wires placed side-by-side with current flowing through them will either attract or repel depending on their orientation due to their magnetic fields interacting
  - A magnetic field can exert a force on a current
- This follows Newton's 3rd Law

$$\vec{F}_{CM} = -\vec{F}_{MC}$$

MEASURING MAGNETIC FIELDS

- From observation, we can see that the magnetic force on a charged particle is directly proportional to the magnetic field, the velocity of the charged particle, and the magnitude of the charge $(F_M \propto B, v, q)$
- We also observe that
  - The magnetic force is greatest when the velocity and magnetic field are perpendicular
  - The magnetic force vanishes when the velocity is in line with the magnetic field

MEASURING MAGNETIC FIELDS – CONT.

- This indicates that the magnetic force is proportional to the sign of the angle between the velocity and magnetic field
- When a charge move perpendicular to a magnetic field, $\theta = 90°$ and $\sin 90° = 1$
  - This produces the maximum force
- When a charge moves in line with the magnetic field, $\theta = 0°$ or $\theta = 180°$, and $\sin 0° = \sin 180° = 0$
  - The force is zero

MAGNETIC FORCE

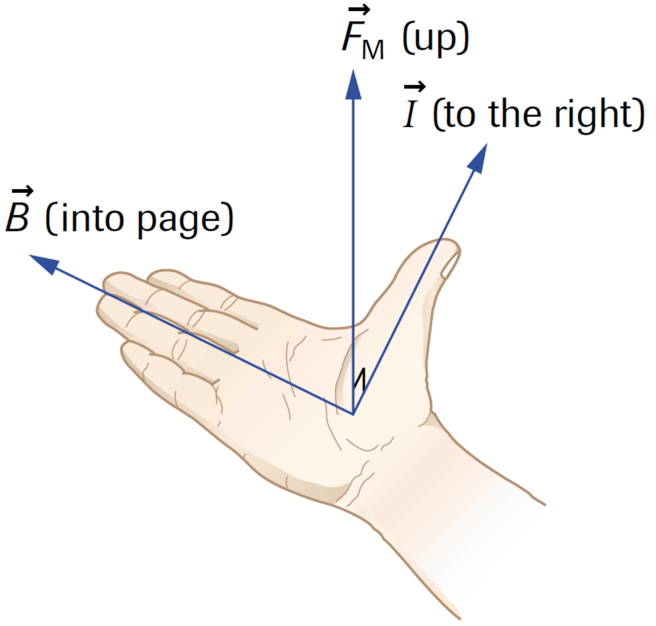
- **Recall: Magnetic Field ($\vec{B}$) [T = 1 kg/C s]:** the field produced by a magnet
- **Magnetic Force ($\vec{F}_M$) [N]:** the force caused by a magnetic field on a charged particle

$$\boldsymbol{F_M = qvB \sin \theta}$$

  - q – charge of the particle [C]
  - v – speed of the particle [m/s]
  - B – magnetic field strength [T]
  - θ – angle between the magnetic field and velocity [°]

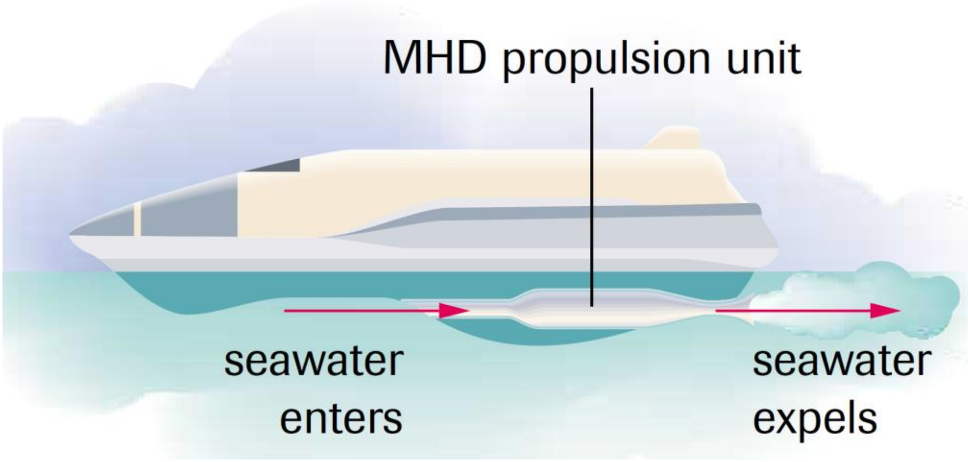
RIGHT-HAND RULE – MAGNETIC FORCE

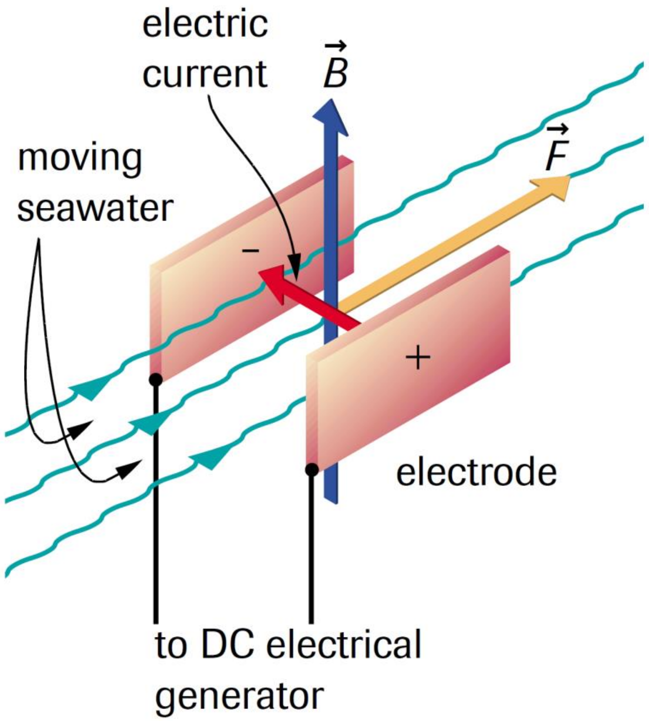
- **Recall:** current moves opposite to e^- flow
- Thumb points in the direction motion of a positive charge (direction of current)
  - For a negative charge, point opposite the direction of motion (direction of current)
- Extended fingers point in the direction of the magnetic field
- The force is in the direction in which your palm would push



APPLICATION: MAGNETOHYDRODYNAMIC PROPULSION

- **Magnetohydrodynamic Propulsion (MHD):** exerts a magnetic force on a current





TRAJECTORY OF A PARTICLE IN A MAGNETIC FIELD

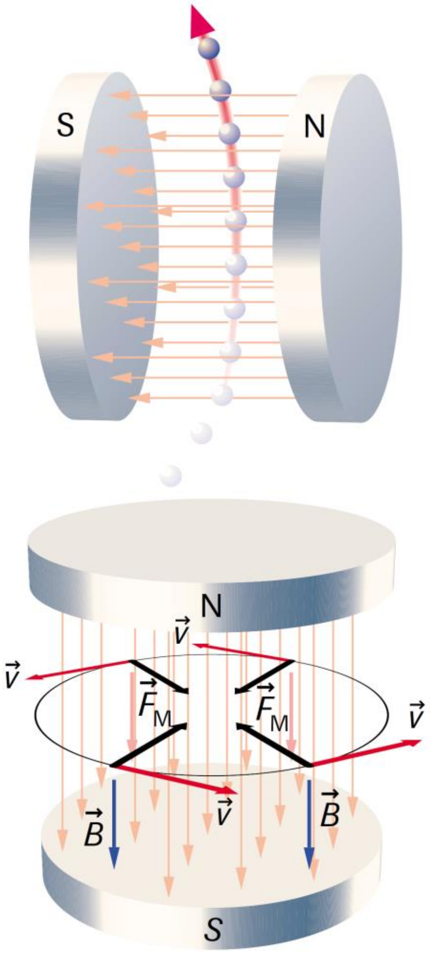
- Consider how $\vec{F}_M$ is always perpendicular $\vec{v}$
- $\vec{F}_M$ will deflect $\vec{v}$ (change the direction)
- $\vec{F}_M$ does not affect the magnitude (speed) of $\vec{v}$ since it does no work on $\vec{v}$; $\cos 90° = 0$
- This means the magnetic force does not change the energy of the particle, only direction

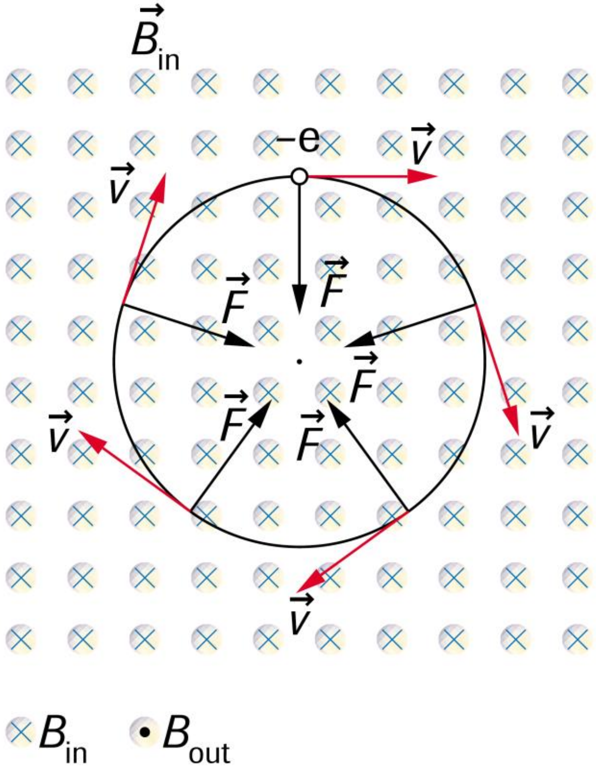
TRAJECTORY OF A PARTICLE IN A MAGNETIC FIELD – CONT.

- If magnetic force is the only force acting on a charged particle, then a particle moving at a constant speed in a uniform magnetic field should move in a circle, and

$$\Sigma F = F_c = F_M$$

- The magnetic force is the centripetal force in this ideal case





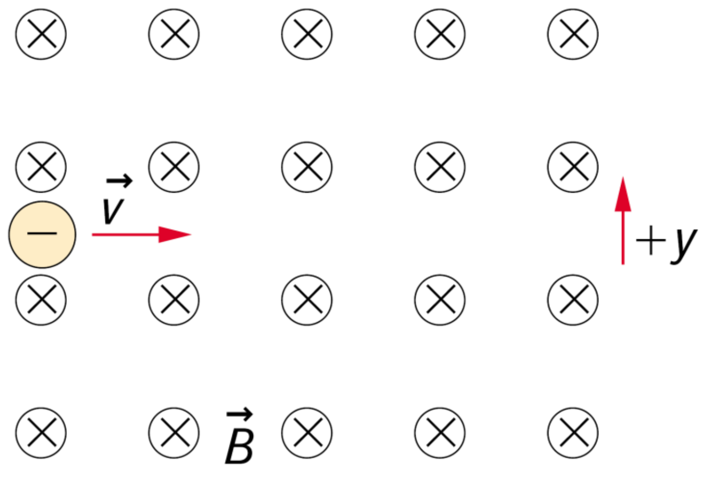
PROBLEM 1

An electron accelerates from rest in a horizontally directed electric field through a potential difference of 46 V. The electron then leaves the electric field, entering a magnetic field of magnitude 0.20 T directed into the page (**Figure 7**).

(a) Calculate the initial speed of the electron upon entering the magnetic field.

(b) Calculate the magnitude and direction of the magnetic force on the electron.

(c) Calculate the radius of the electron's circular path.



PROBLEM 1 – SOLUTIONS

$\Delta V = 46\text{ V}$ $\quad v = ?$

$B = 0.20\text{ T} = 0.20\text{ kg/C}\cdot\text{s}$ $\quad F_M = ?$

$m_e = 9.11 \times 10^{-31}$ kg (from Appendix C) $\quad r = ?$

$q = 1.6 \times 10^{-19}\text{ C}$

(a) The electric potential energy lost by the electron in moving through the electric potential difference equals its gain in kinetic energy:

$$-\Delta E_E = \Delta E_K$$

$$q\Delta V = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

$$= \sqrt{\frac{2(1.6 \times 10^{-19}\text{ C})(46\text{ V})}{9.11 \times 10^{-31}\text{ kg}}}$$

$$v = 4.0 \times 10^6\text{ m/s}$$

The initial speed of the electron upon entering the magnetic field is 4.0×10^6 m/s.

PROBLEM 1 – SOLUTIONS

(b)

$$F_M = qvB\sin\theta$$

$$= (1.6 \times 10^{-19}\ \text{C})(4.0 \times 10^{6}\ \text{m/s})(0.20\ \text{kg/C·s}) \sin 90°$$

$$F_M = 1.3 \times 10^{-13}\ \text{N}$$

The magnitude of the force is 1.3×10^{-13} N.

To apply the right-hand rule, point your right thumb in the direction opposite to the velocity, as required for a negative charge. Point your fingers into the page and perpendicular to it. Your palm now pushes toward the bottom of the page. Therefore, $\vec{F}_M = 1.3 \times 10^{-13}$ N [down].

PROBLEM 1 – SOLUTIONS

(c) Since the magnetic force is the only force acting on the electron and it is always perpendicular to the velocity, the electron undergoes uniform circular motion. The magnetic force is the net (centripetal) force:

$$F_M = F_c$$

$$qvB = \frac{mv^2}{r} \qquad \text{(since } \sin 90° = 1\text{)}$$

or $$r = \frac{mv}{Bq}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(4.0 \times 10^{6}\ \text{m/s})}{(0.20\ \text{T})(1.6 \times 10^{-19}\ \text{C})}$$

$$r = 1.1 \times 10^{-4}\ \text{m}$$

The radius of the circular path is 1.1×10^{-4} m.

CHARGE-TO-MASS RATIOS

- Recall: a cathode ray tube uses the large potential difference between the cathode and anode to accelerate an e^-
- J.J. Thomson (1856-1940) used cathode ray tubes to send beams of electrons through parallel plates and coils
- In all measurements with different strengths and orientations of fields, the deflection of the beam was consistent with a negatively charged particle
  - Thomson named this particle the electron

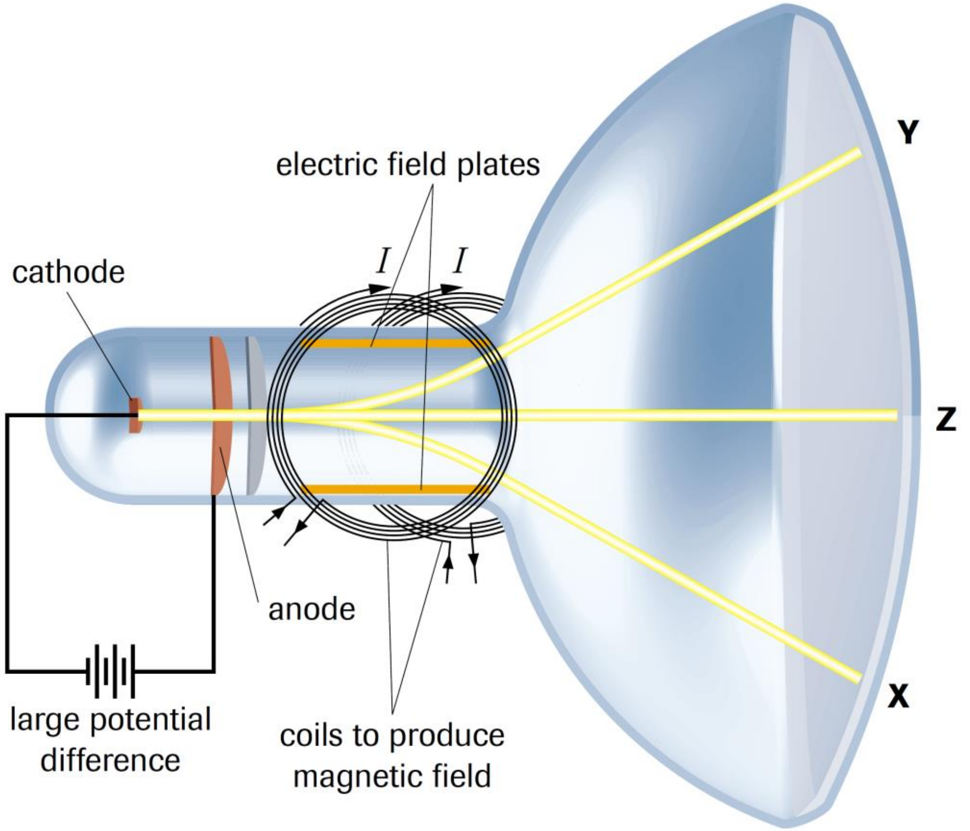
CHARGE-TO-MASS RATIOS – CONT.

- A current I through the coils generates a magnetic field of magnitude B
- The field deflects the e^- along a circular arc of radius r towards X
- Recall: $F_M = F_c$

$$qvB\sin\theta = \frac{mv^2}{r}$$

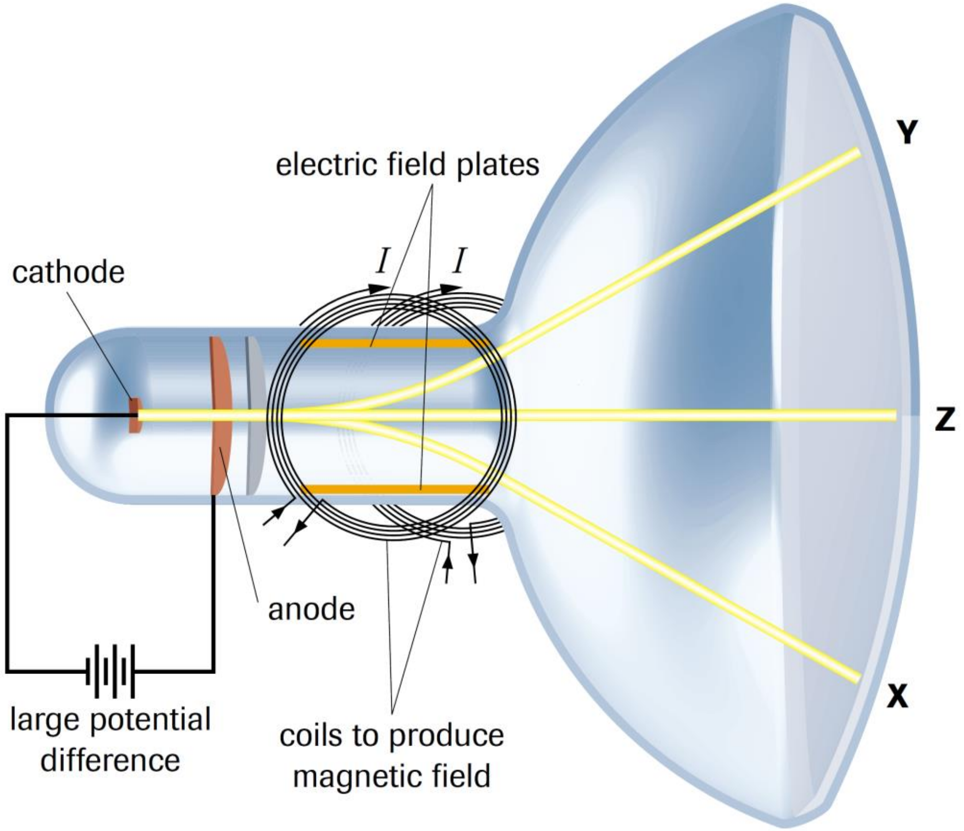
$$evB\sin 90° = \frac{mv^2}{r}$$

$$\frac{e}{m} = \frac{v}{Br}$$



CHARGE-TO-MASS RATIOS – CONT.

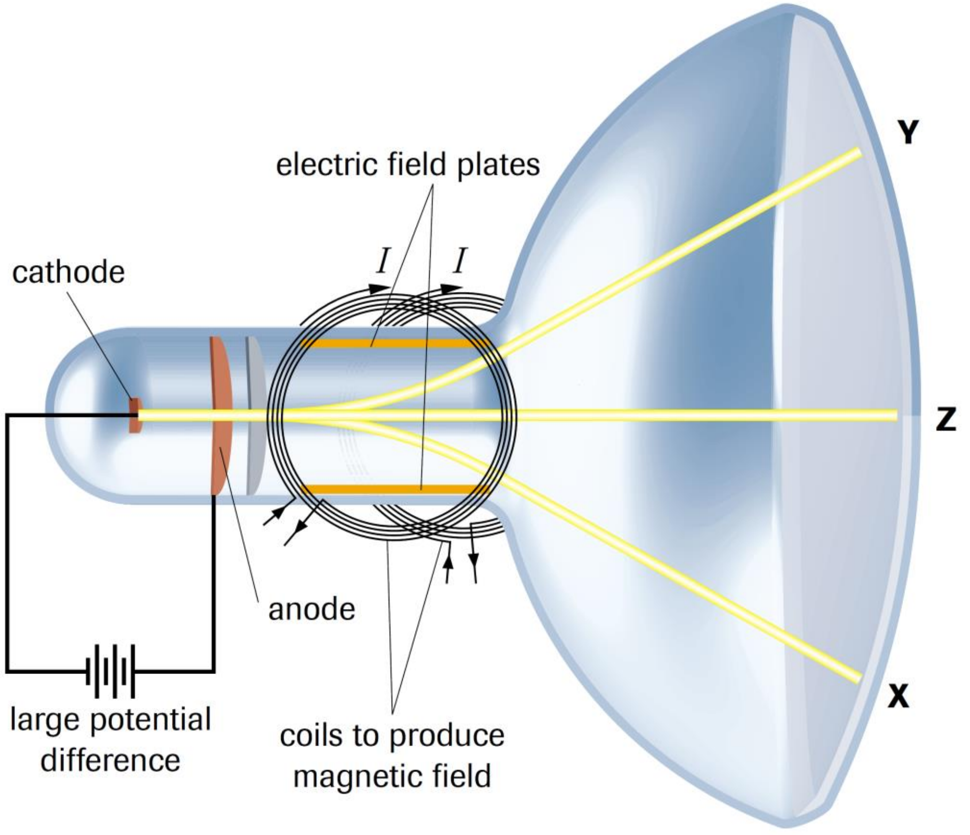
- Turning off the coils and generating a potential difference in the parallel plates (lower plate negative) deflects the electrons towards Y
- Note: at typical laboratory speeds, force of gravity is negligible



CHARGE-TO-MASS RATIOS – CONT.

- Balancing the electric and magnetic forces allows for the straight path, Z

$$F_M = F_E$$
$$qvB = q\varepsilon$$
$$evB = e\varepsilon$$
$$v = \frac{\varepsilon}{B}$$



CHARGE-TO-MASS RATIOS – CONT.

- From before, with $v = \frac{\varepsilon}{B}$,

$$\frac{e}{m} = \frac{v}{Br}$$

$$\frac{e}{m} = \frac{\varepsilon}{B^2 r}$$

- The accepted value for the ratio of charge to mass of an electron is

$$\boldsymbol{\frac{e}{m} = 1.76 \times 10^{11}\ \mathrm{C/kg}}$$

CHARGE-TO-MASS RATIOS – CONT.

- More generally, the charge-to-mass ratio is

$$\frac{q}{m} = \frac{\varepsilon}{B^2 r}$$

  - q – charge of the particle [C]
  - m – mass of the particle [kg]
  - ε – electric field [N/C]
  - B – magnetic field [T]
  - r – radius of curvature of charge [m]

PROBLEM 2

Calculate the mass of chlorine-35 ions, of charge 1.60×10^{-19} C, accelerated into a mass spectrometer through a potential difference of 2.50×10^{2} V into a uniform 1.00-T magnetic field. The radius of the curved path is 1.35 cm.

PROBLEM 2 – SOLUTIONS

$q = 1.60 \times 10^{-19}$ C

$\Delta V = 2.50 \times 10^{2}$ V

$B = 1.00$ T $= 1.00$ kg/C·s

$r = 1.35$ cm $= 1.35 \times 10^{-2}$ m

$m = ?$

From $\Delta E_c = \Delta E_K$ and $F_M = F_c$, we have the following two equations:

$$qvB = \frac{mv^2}{r} \quad \text{and} \quad \frac{1}{2}mv^2 = q\Delta V$$

PROBLEM 2 – SOLUTIONS

Isolating *v* in both

$$v = \frac{qBr}{m} \quad \text{and} \quad v = \sqrt{\frac{2qV}{m}}$$

Equating the two expressions for the speed:

$$\frac{qBr}{m} = \sqrt{\frac{2q\Delta V}{\mathrm{m}}}$$

PROBLEM 2 – SOLUTIONS

Squaring both sides:

$$\frac{q^2B^2r^2}{m^2} = \frac{2q\Delta V}{m}$$

$$m = \frac{qB^2r^2}{2\Delta V}$$

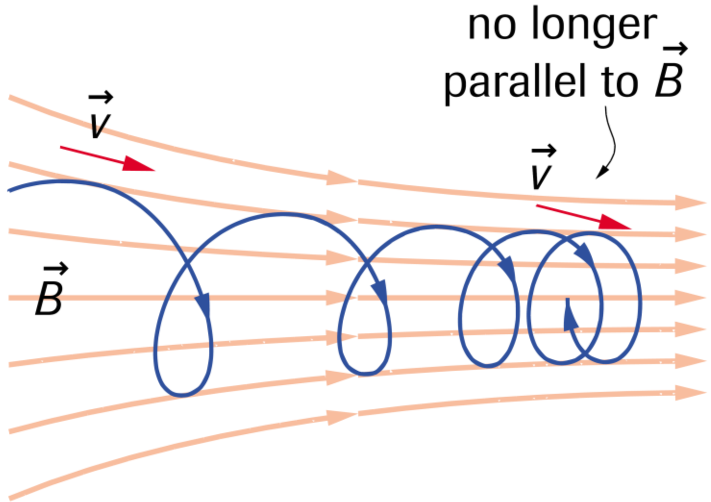
$$= \frac{(1.60 \times 10^{-19}\ \text{C})(1.00\ \text{kg/C}\cdot\text{s})^2\ (1.35 \times 10^{-2}\ \text{m})^2}{2(2.50 \times 10^2\ \text{V})}$$

$$m = 5.83 \times 10^{-26}\ \text{kg}$$

The mass of the chlorine-35 ions is 5.83×10^{-26} kg.

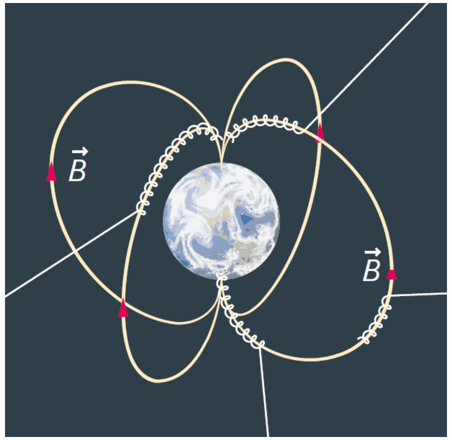
EFFECTS OF MAGNETIC FIELDS

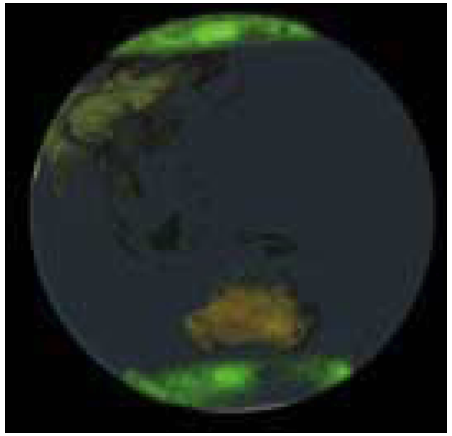
- When e^- are not travelling directly perpendicular to the magnetic field:
  - the perpendicular component still travels in a circle
  - The parallel component continues its path
- This results in a spiral
- A non-uniform, increasing magnetic field will slow down the path of the e^- by reducing the component moving forward
  - It can even make it go in reverse! This is called a magnetic mirror



EFFECTS OF MAGNETIC FIELDS – CONT.

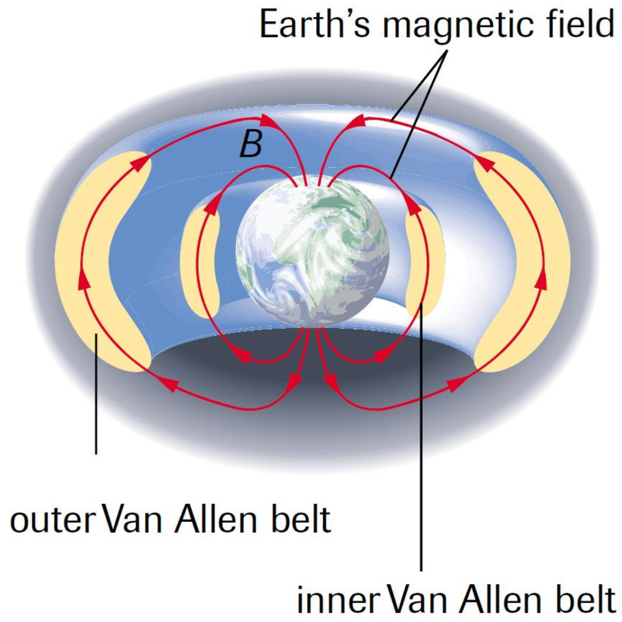
- Charged particles from the Sun’s rays spiral around the magnetic field lines of the Earth
- This prevents the particles from reaching areas like the equator and they build up at the poles
- This results in the aurora borealis (northern lights) and the aurora australis (southern lights)





EFFECTS OF MAGNETIC FIELDS – CONT.

- There are two belts of charged particles trapped in Earth's magnetic field
- These emit such strong radiation that all spacecraft avoid them
- These particles are usually deflected away from the stronger fields of the poles



DERIVING THE EQUATION FOR THE MAGNETIC FORCE

- Recall: for a point charge, magnetic force is
$$F_M = qvB\sin\theta$$
- Considering a group of n charges, we get a net force of
$$F_M = n(qvB\sin\theta)$$
- This gives us enough information to solve for the force on a conductor, but some of the values are difficult to measure
- Since we can measure current more easily, we want to derive an equation involving current rather than charge

DERIVING THE EQUATION FOR THE MAGNETIC FORCE – CONT.

- If n charged particles with charge q pass a point in a conductor in time Δt,

$$I = \frac{nq}{\Delta t}$$

- So our charge can be represented by

$$q = \frac{I\Delta t}{n}$$

DERIVING THE EQUATION FOR THE MAGNETIC FORCE – CONT.

- It is also difficult to measure the speed of the particles, but we can easily determine the length of the conductor
- Since $v = \frac{d}{\Delta t}$, where d is the lengths of the conductor l,

$$v = \frac{l}{\Delta t}$$

- Combining our equations for q and v with F_M, we get

$$F_M = nqvB \sin\theta$$

$$F_M = n\left(\frac{I\Delta t}{n}\right)\left(\frac{l}{\Delta t}\right)B \sin\theta$$

$$\boldsymbol{F_M = IlB \sin\theta}$$

MAGNETIC FORCE ON A CONDUCTOR

- Magnetic force on a conductor:

$$\boldsymbol{F_M = IlB \sin\theta}$$

  - I – electric current [A]
  - l – length of conductor [m]
  - B – magnetic field [T]
  - θ – angle between current and magnetic field [°]
- Important Observations:
  - The force on the conductor F is in a direction perpendicular to both the magnetic field B and the direction of the current I
  - Reversing either the current direction or the magnetic field reverses the direction of the force.

MAGNETIC FORCE ON A CONDUCTOR – CONT.

- To find the base units of the tesla, let's isolate B

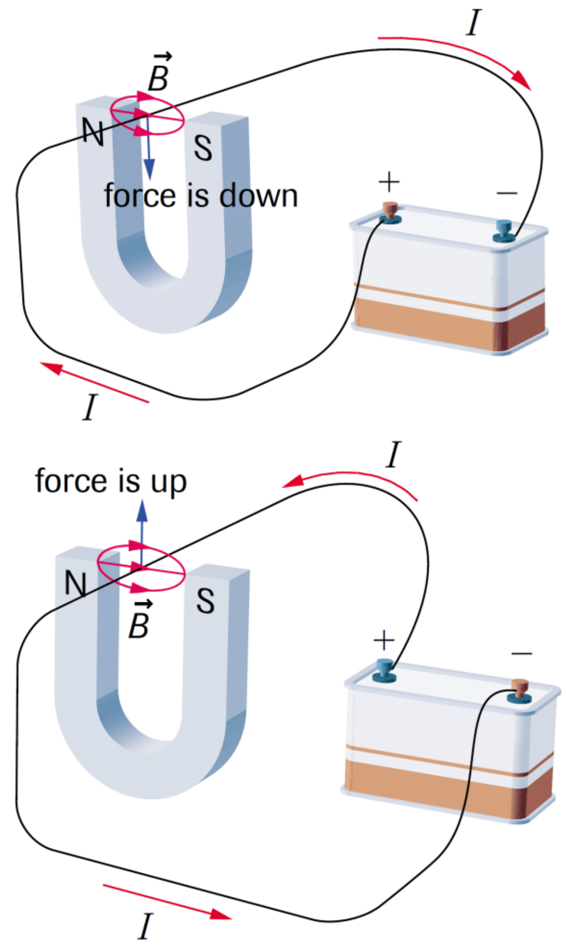
$$B = \frac{F_M}{Il\sin\theta}$$

$$[\mathrm{T}] = \frac{[1\ \mathrm{N}]}{[1\ \mathrm{A}][1\ \mathrm{m}][1]}$$

- So $[\mathrm{T} = \mathrm{N/A\ m}]$

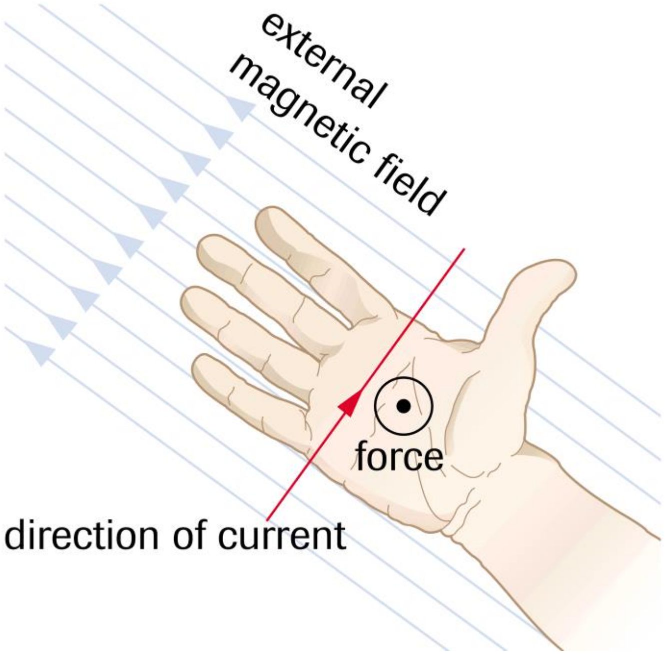
MAGNETIC FORCE ON A CONDUCTOR – CONT.

- For a horseshoe magnet (or two opposite poles) we have a uniform magnetic field between the poles
- For a current run perpendicular to the magnetic field, we can control the magnetic force by switching the direction of the current



RIGHT-HAND RULE
THE MOTOR PRINCIPLE

- **Right-Hand Rule for the Motor Principle:** If the right thumb points in the direction of the current (flow of positive charge), and the extended integers point in the
- direction of the magnetic field, the force is in the direction in which the right palm pushes.



PROBLEM 3

A straight conductor 10.0 cm long with a current of 15 A moves through a uniform 0.60-T magnetic field. Calculate the magnitude of the force on the conductor when the angle between the current and the magnetic field is (a) 90°, (b) 45°, and (c) 0°.

PROBLEM 3 – SOLUTIONS

$I = 15\text{ A}$ $\quad B = 0.60\text{ T}$

$l = 10.0\text{ cm}$ $\quad F = ?$

In the general case, the magnitude of the force is given by

$$F = IlB \sin\theta$$
$$= (15\text{ A})(0.60\text{ T})(0.10\text{ m}) \sin\theta$$
$$F = (0.90\text{ N}) \sin\theta$$

(a) when $\theta = 90°$, $\sin\theta = 1$ and $F = 0.90\text{ N}$

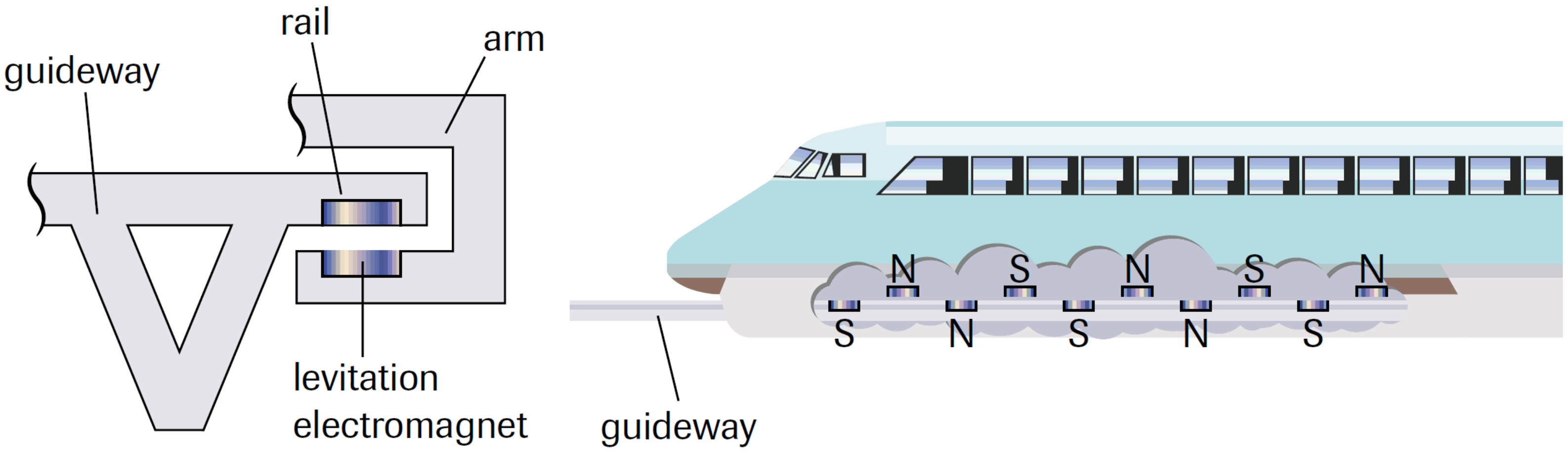
(b) when $\theta = 45°$, $\sin\theta = 0.707$ and $F = (0.90\text{ N})(0.707) = 0.64\text{ N}$

(c) when $\theta = 0°$, $\sin\theta = 0$ and $F = 0\text{ N}$

The magnitude of the force is 0.90 N at $\theta = 90°$, 0.64 N at $\theta = 45°$, and 0 N at $\theta = 0°$. In each case, the direction of the force is given by the right-hand rule for the motor principle.

APPLICATION: MAGLEV TRAINS

- Currents control the poles of the magnets on the train and track
- Attraction and repulsion forces between the magnets propel the train forward; maximum speed is much higher than normal trains
- Currents are reversed to slow down (reversing the poles of the magnets)



SUMMARY – MAGNETIC FORCE ON MOVING CHARGES

- A current can exert a force on a magnet, and a magnet can exert a force on a current.
- $F_M = QvB \sin\theta$
- The direction of the magnetic force is given by the right-hand rule.
- The speed of an electron in a cathode-ray tube can be determined with the help of magnetic deflecting coils and electric deflecting plates. The same apparatus then gives the charge-to-mass ratio of the electron. Combining this determination with the charge of an electron from the Millikan oil-drop experiment yields the mass of the electron.

SUMMARY – MAGNETIC FORCE ON A CONDUCTOR

- The magnitude of the force on the conductor F is in a direction perpendicular to both the magnitude of the magnetic field B and the direction of the current I: in SI units, $F = IlB \sin\theta$.
- Reversing either the current direction or the magnetic field reverses the direction of the force.

PRACTICE

Readings

- Section 8.2 (pg 392)
- Section 8.3 (pg 404)

Questions

- pg 402 #1-8
- pg 407#1-5